APPENDIX 1. MEASUREMENT OF FLOWS

Bernoulli's Equation

\[ \frac{1}{2} (p_2 - p_1) + \frac{u_2^2 - u_1^2}{2} + g (z_2 - z_1) = 0 \]

\[ p_1 + \frac{1}{2} \rho u_1^2 = p_2 = \frac{1}{2} \rho u_2^2 = p_0 \]

- \( p_0 \) = static pressure
- \( p_0 \) = stagnation pressure
- \( u \) = velocity

\[ p_s + \frac{1}{2} \rho u^2 = p_0 \]

\[ U = \left( \frac{2 (p_0 - p_s)}{\rho} \right)^{1/2} \]

measure \( \Delta p \) to determine air velocity
Hot Wire Anemometer

![Diagram of Hot Wire Anemometer](image)

**Figure 2**: Tungsten Hot Wire Sensor and Support Needles - 0.00015" Dia. (0.0038 mm)

Fast Response - measure turbulence

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Hot Film

![Diagram of Hot Film Sensor](image)

**Figure 3**: Cylindrical Hot Film Sensor and Support Needles - 0.002" Dia. (0.051 mm)

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Heat transfer from a hot wire \( q = a + b \sqrt{u} (T_w - T_a) \)

Electrical heating of wire \( q = I^2 R_w \)

\[ I^2 R_w(1 + a \rho (T_w - T_a)) \]

Resistor of wire is a function of \( T \)

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**a)** Constant \( T \) hot wire \( (T_w = \text{constant}) \)

Power to wire: \( P = f(u) \)

**b)** Constant current hot wire \( (I = \text{constant}) \)

\( T_w = f(u) \)
**OBSTRUCTION METERS**

![Diagram of obstruction meters](image)

**Figure 15.3** (a) A venturi. (b) A flow-nozzle. (c) An orifice flowmeter.

Continuity equ.
\[ \frac{A_1}{A_2} u_1 = \frac{A_2}{A_1} u_2 \]

Bernoulli & continuity:
\[ \frac{1}{2} u_2^2 - \frac{1}{2} u_1^2 \left( \frac{A_1}{A_2} \right)^2 = p_1 - p_2 = \Delta p \quad \text{if} \quad \frac{p_1}{p_2} = \frac{\Delta p}{p_2} \]

Static pressures

\[ u_2 = \sqrt{\frac{\Delta p}{2}} \sqrt{\frac{1}{1 - \left( \frac{A_1}{A_2} \right)^2}} \]

\[ \phi = A_2 u_2 = A_2 \sqrt{\frac{1}{1 - \left( \frac{A_1}{A_2} \right)^2}} \sqrt{\frac{\Delta p}{2}} \quad \text{ideal} \]

\[ \phi = \frac{V P A_2}{v_1} \frac{1}{\left( \frac{A_1}{A_2} \right)^2} \sqrt{\frac{2 \Delta p}{V}} \quad \text{discharge coefficient} \]

\[ V \approx 0.58 \text{ venturi}; \quad \approx 0.96 \text{ orifice} \]

\[ \phi \propto \sqrt{\Delta p} \quad \text{compressibility factor} \]

\[ \phi = 1 \text{ for incompressible fluid} \]


**Figure 7.34 Rotameter.**

Drag force = Effective wt. of float:

\[ C_d \left( \frac{1}{2} \rho A^2 \right) \left( \frac{4}{3} \pi d^3 \right) = ( \rho_a - \rho_t ) V g \]

\[ C_d = \text{coef of drag} \]
\[ d = \text{diam. of float} \]
\[ \rho_a = \text{fluid density inside} \]
\[ V = \text{float volume} \]
\[ d = \text{diam. of tube} \]

\[ \rho = f( \alpha ) = \text{by calibration} \]
\[ \rho = f( \alpha ) \] but not so

- for sharp edged float

- can be shown that

\[ Q \alpha \frac{\rho_a^{1/2}}{a} \alpha \frac{\rho_i}{\rho_a} \]

\[ c = \text{inside tube} \]
\[ a = \text{ambient} \]

**Effect of Pa**

if float in same position (same \( \rho_i \)) ambient at calibration

\[ Q_{\text{true}} = Q_{\text{cal}} \frac{\rho_a}{\rho_i} \text{ambient} \]

**Effect of gas \( q \) (or \( \rho_i \))**

\[ Q_{\text{true}} = Q_{\text{cal}} \sqrt{\frac{\text{true}}{\text{ideal}}} = Q_{\text{cal}} \sqrt{\frac{\text{true}}{\text{ideal}}} \]

\[ \text{pressures inside rotameter} \]

\[ \text{ambient pressure} \]

\[ \text{true} = \text{ideal} \]

\[ \text{true} \]

\[ \text{ideal} \]
Critical Orifice

\[ Q_a = \frac{0.58kA_a}{g_a} \left( \frac{\gamma}{\gamma - 1} \right)^{\frac{1}{2}} \left( \frac{p_1}{p_1} \right)^\frac{5}{2} \]

\[ A_a - \text{orifice area} \]
\[ \gamma = 1.4 \text{ air} \]
\[ k - \text{discharge coeff.} \]

For gas \( P_i = g_1RT_i \):
\[ Q_a = \frac{0.58kA_a}{g_a} \left( \frac{\gamma}{\gamma - 1} \right)^{\frac{1}{2}} \left( \frac{1}{\gamma} \right) \frac{1}{P_i} \]

**Example 1:**

\[ Q_a \text{ vac.} \]
\[ \text{c.o.} \]

If \( P_i = P_i \) & \( P_e = g_3RT_a \)

\[ Q_a \neq f (P_i \text{ or } g_3) \]

\[ \Rightarrow \text{constant } Q \] that is a weak function of \( T \)

**Example 2:**

\[ Q_a \text{ filter} \]

\[ \text{Vac.} \text{ pump} \]

\( Q_a \) does depend on \( P_i \), but if the pressure drop is small \( (P_a \approx P_i) \), where pressures are absolute, then, as Example 1, \( Q_a \approx \) not a function of \( P_i \)
LAMINAR FLOW METER

constant diameter tube

Laminar flow: \( \text{Re} = \frac{\rho u D}{\mu} \leq 2000 \)

- \( \rho \) - gas density
- \( u \) - velocity in tube
- \( D \) - tube diameter
- \( \mu \) - gas viscosity

Entrance length, \( L_e \)

\[ L_e \propto (0.06 \text{Re}) 2r \]

Hagen-Poiseuille flow

Calibrate:

\[ \Delta P = \frac{8 \mu L Q}{\pi r^4} \]

\( \mu \) is a function of \( P \) from K.T.G.

\( \mu \) is a function of \( P \)
POSITIVE DISPLACEMENT METERS

- Often used to calibrate other flow meters

Figure 2.13 Soap bubble spirometer.

Figure 2.12 Spirometer.

COMMERCIAL AVAILABLE "Calibrator"

- In all 3 cases, must account for added H2O vapor.
- Use ideal gas law, assume RH in water is 100%.
- RH enklin - ?

Others include:
- Gas meter
- Dry Cal
Mass Flow Meters

E.g., McMillan Company model based on thermal sensing technology. Flow enters the unit and a portion is redirected into a small tube. This tube has two coils, one downstream from the other. Each coil is heated, and, as the gas passes through the tube, the electronics sense the amount of heat transferred from one coil to the other. The output (voltage) this sensor is directly related to the specific heat \( (c_p/m) \) of the gas being measured. If a different gas is measured then the calibration gas, a correction factor related to the specific heats of the calibration and measured gases must be applied.